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A STUDY OF THE APPARENT CHANGE IN THE FOUNDATION RESPONSE OF A NINE-STORY REINFORCED CONCRETE BUILDING

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ABSTRACT

Forced vibration tests of the Millikan Library building, a nine-story reinforced concrete shear-wall structure, were conducted in 1966 and 1967, before the San Fernando earthquake, and again in 1974. The measured foundation response of the structure reported for the two tests was significantly different: in the earlier tests, motion of the foundation in the N-S direction contributed only about 3 per cent to the total roof motion, whereas in the more recent tests almost 30 per cent of the roof motion was contributed by foundation compliance. A lengthening of the fundamental period of vibration of 11 per cent was also noted.

The purpose of this study is to examine the indication that the foundation response of the structure may have changed because of the earthquake. To determine whether the observed changes in foundation response are consistent with the change in natural period, two analytical models of the Millikan Library building were developed. Both of these models include the effects of foundation compliance and one includes the effects of shear deformations in the walls of the structure. The results of these simple analyses show the changes of mode shape and period observed between the two tests to be consistent.

From the analysis, and from an examination of what is thought to be minor earthquake damage at the ground floor level of the structure, the authors conclude that the most probable cause of the differences observed in the two tests is the loss of rotational and translational stiffness provided by retaining walls, concrete slabs and other stiff, but brittle elements.

INTRODUCTION

The Millikan Library building is a nine-story reinforced concrete shear-wall structure located on the campus of the California Institute of Technology in Pasadena, California. Due to its convenient location the structure has been the subject of numerous forced vibration tests by students and faculty of Caltech over the last 10 years (Kuroiwa, 1967; Kuroiwa and Jennings, 1968; Foutch *et al.*, 1975; Foutch, 1976).

The first vibration tests of Millikan Library were conducted in 1966 and 1967 by Kuroiwa and Jennings during (and shortly after) construction of the building. One feature of this study was the measurement of the motion of several points in the basement and on the ground outside of the building during forced excitation of the structure at resonance. This provided information concerning the amount of soil-structure interaction that might be expected for this type of structure.

In 1974 a second, more extensive, series of tests were conducted by Foutch *et al.* (1975). During these tests, three-dimensional motions were measured for 50 points on each of 6 floors including the basement, as well as at approximately 100 points on the ground outside of the structure. The data were taken for shaking in both the N-S and E-W fundamental modes of vibration. These measurements quantified the interaction of the various load-resisting systems of the building and provided comparative data for theoretical soil-structure interaction studies. (Luco *et al.*, 1975; Wong, 1975)

A comparison of results of the two investigations suggests the unexpected result that the foundation response of the structure, particularly in the stiffer N-S direction, changed significantly in the interim between the two tests, which included the San Fernando earthquake. The measurements by Kuroiwa and Jennings (1968) indicate that only about 3 per cent of the roof motion was attributable to motion at the base of the structure during vibration in the first N-S mode. Those by Foutch *et al.* (1975), however, indicate that almost 30 per cent of the roof motion is due to foundation compliance. Since the acceleration levels in the building reached 35 per cent g at the top during the San Fernando earthquake and 20 per cent g at the base, it seems possible that the differences in the two sets of test results may be attributable to a change in the foundation resistance caused by the earthquake. It is also possible that experimental error occurred during one of the tests. The purpose of the present study is to examine the results reported in these two tests and to attempt to clarify the substantial difference reported for the amount of soil-structure interaction.

Description of the Building and Instrumentation

The Robert A. Millikan Memorial Library is a nine-story reinforced concrete shear-wall structure. The building is 69 by 75 feet in plan and stands 144 feet above grade and 158 feet above the basement level. The majority of the lateral loads in the transverse (N-S) direction are resisted by 12-in reinforced concrete shear walls located at the east and west ends of the structure. In the longitudinal (E-W) direction, the primary resistance to lateral loads is provided by the 12-in reinforced concrete walls of the central core which houses two elevators and an emergency stairway. The foundation system is composed of a central pad 32 ft wide by 4 ft deep that extends between the east and west curved shear walls. In addition to this, continuous foundation beams 10 ft wide by 2 ft deep run east-west beneath the rows of columns at the north and south ends of the building; these are connected to the central pad by stepped beams. Plan and section views of the foundation system are shown in Figure 1a. Similar views of the structural system are shown in Figure 1, b and c, respectively. A more detailed description of the building may be found in Kuroiwa (1967).

A six-channel accelerometer-amplifier-recorder system was used by Kuroiwa to measure the response of the building and nearby ground. The later study by Foutch employed four Ranger seismometers with the associated signal conditioning and recording systems as basic instrumentation. Complete descriptions of the instrumentation used by Kuroiwa and Foutch may be found in their respective reports (Kuroiwa, 1967; Foutch, 1976).

Test Results

The vibrational properties of the Millikan Library building for motion in the first N-S mode as determined by Kuroiwa and by Foutch are shown in Figure 2, a and b, respectively. Kuroiwa's results indicate that 2 per cent of the roof translation is caused by translation at the base of the structure and less than 1 per cent is caused by rotation at the basement level. These results differed substantially from those reported by Foutch who determined that 4 per cent of the roof translation was due to base translation and 25 per cent was due to base rotation. In terms of stiffnesses, the results imply that the rotational stiffness of the foundation was reduced by a factor of about 30 between the times that the two tests were conducted, and that a much smaller change in translational stiffness occurred. It is known from Rayleigh's principle that small changes of natural frequency can be associated with much larger changes in the mode shape. This fact is used to obtain relatively accurate estimates

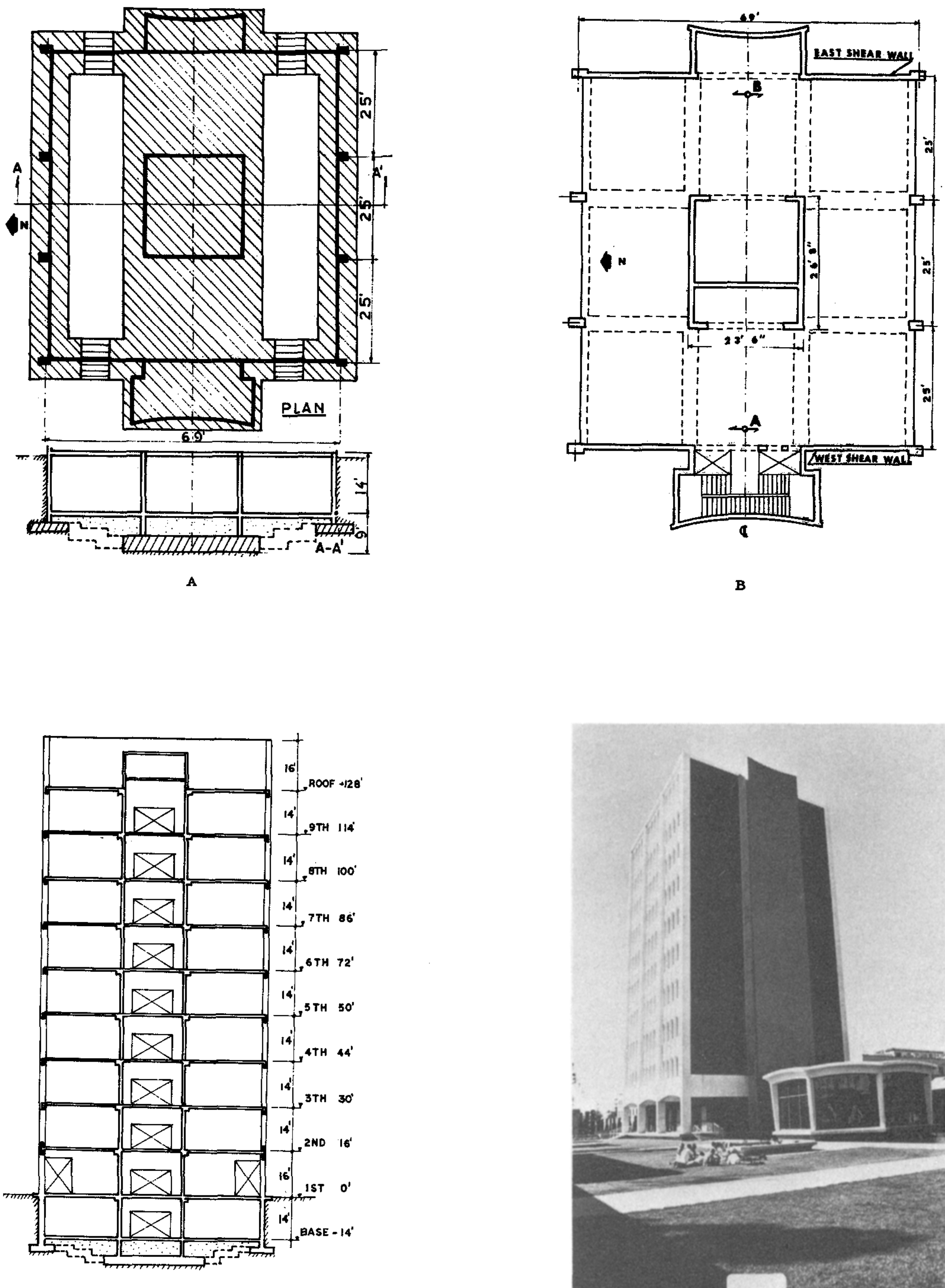


FIG. 1. Millikan Library Building: (a) foundation plan and N-S section; (b) typical floor plan; (c) N-S section; (d) view of building looking Northwest.

of natural periods from relatively poor approximations of the fundamental mode shape. It was not clear in this instance, however, whether the amount of change observed in the frequency of vibration was large enough to be consistent with the observed change in the mode shape.

The largest difference in the two tests occurs in the amount of vertical motion measured at the outer portions of the building. The second test indicated about 25 times the motion measured in the first tests. A small fraction of the difference can be explained by the fact that the measurements of rocking made by Kuroiwa were made at the point of minimum vertical motion of the base. The second, more extensive measurements, (Foutch *et al.*, 1975) revealed bending of the basement slab with maximum vertical motion occurring at the north and south ends of the shear walls. The minimum vertical deflections along the north and south edges of the slab occurred on the centerline of the building and were about half the maximum. The average of

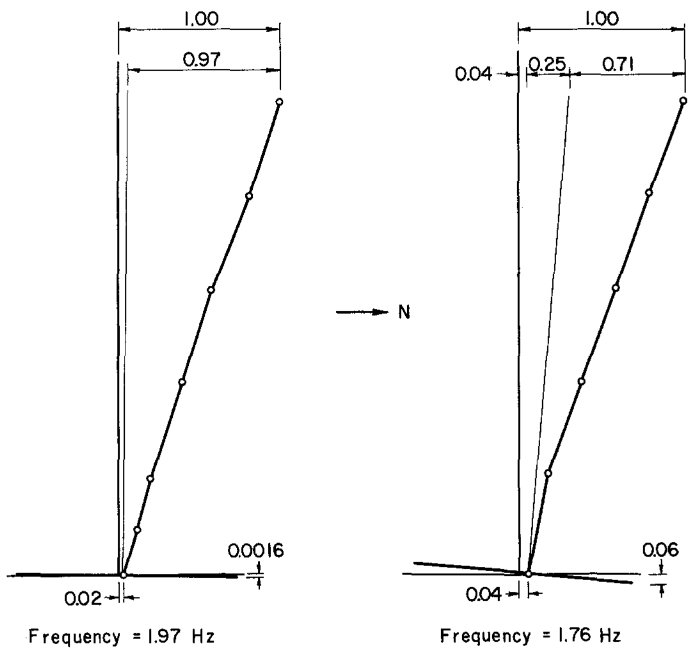


FIG. 2. Contribution of foundation deformation to roof motion for N-S shaking: (a) Kuroiwa and Jennings as measured in 1967; Foutch *et al.* as measured in 1974.

these vertical motions was used by Foutch and is shown in Figure 2b. For purposes of comparison, the rotational components of Kuroiwa's mode shape was adjusted accordingly from that which he reported. This is the result shown in Figure 2a.

The vertical motions recorded in the first tests were quite small, near the limits of sensitivity of the equipment. The four vertical motions used to define the rocking are, however, consistent in both the N-S and E-W motions. This indicates that the small signal-to-noise ratio is not a likely source for a discrepancy of the size sought. If there is an experimental error in the first series of tests, it seems most likely that the source would be an insufficient amplification of the output of the transducer. There are two main arguments against this possibility. First, the same instrumentation was used in a number of vibration tests before and after the Millikan tests without any reported difficulty. Some of the tests involved forced vibration of dams, where the motions are also very small. Secondly, the linearity of the amplifier was checked before testing and monitored occasionally during the tests by noting the change in signal strength with amplifier setting.

As an additional check on the possibility that the foundation rocking was under-estimated in the first tests, the same transducer-amplifier-recorder system was restored to service and was used to measure the rocking exhibited in May 1977. The motion measured at nine points on the foundation and on the roof indicated that about 22 per cent of the horizontal roof motion is contributed by rocking and 5 per cent by translation of the foundation. This result is consistent with the second series of tests within the accuracy of the measurements, and indicates that if such foundation motions were present in 1966, they could have been measured with the equipment used.

Analysis

In order to determine if the change in the natural frequency of vibration was quantitatively consistent with the changes in the mode shape, two simple models of the

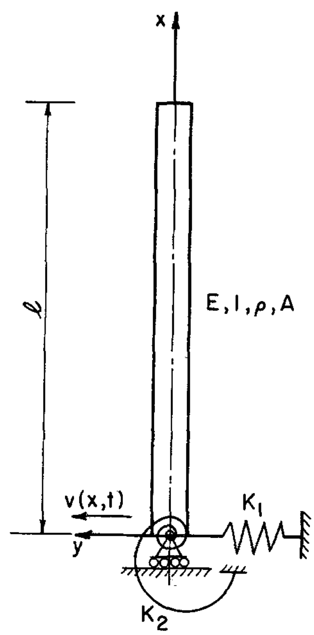


FIG. 3. Euler beam model with a translational spring and a rotational spring at the base. The system models motion at the library in the N-S direction.

building-foundation system were constructed. Since tests indicated that the behavior of the structure in the N-S direction was dominated by the bending of the shear walls at the east and west ends of the building, a cantilevered Euler beam was first chosen to model the structure. Translational and rotational springs were included at the base to model the effects of the foundation deformation. Figure 3 shows a sketch of this model. A second model which includes the effects of shear deformations in the structure is presented below.

From Timoshenko *et al.* (1974), the differential equation of the vibrating beam may be written

$$\frac{\partial^4 v(x, t)}{\partial x^4} = \frac{1}{a^2} \frac{\partial^2 v(x, t)}{\partial t^2} \tag{1}$$

where

$$a^2 = \frac{EI}{\rho A}. \tag{2}$$

In equation (2), E is the modulus of elasticity; I is the moment of inertia of the beam; ρ is the density of the material; A is the cross sectional area of the beam. Equation (1) was solved by separation of variables, which leads to the well-known characteristic function

$$X(x) = C_1 \sin(kx) + C_2 \cos(kx) + C_3 \sinh(kx) + C_4 \cosh(kx) \tag{3}$$

where

$$k^2 = \frac{\omega}{a}. \tag{4}$$

In (4), ω is the frequency of vibration in radians per second.

The characteristic function, equation (3), may be shown (Foutch, 1976) to satisfy the boundary conditions for an Euler beam with a translational and a rotational spring at the base only if the frequency equation,

$$\left(\frac{EI}{l^2}\right)^2 [\cos(\alpha)\cosh(\alpha) - 1]\alpha^4 + \frac{EIk_2}{l^3}[\sin(\alpha)\cosh(\alpha) - \cos(\alpha)\sinh(\alpha)]\alpha^3 + \frac{EIk_1}{l}[\sin(\alpha)\cosh(\alpha) - \cos(\alpha)\sinh(\alpha)]\alpha + k_1k_2[-1 - \cos(\alpha)\cosh(\alpha)] = 0 \tag{5}$$

with $\alpha = kl$, is satisfied.

The first step of this investigation was to construct a model of the building in the N-S direction that would closely match the dynamic characteristics measured by Kuroiwa. To obtain a realistic value for EI, the total weight of the building was assumed to be distributed over the height of the building (142 feet). Using the resulting value for ρA ($\rho Ag = 1.97 \times 10^5$ lb/ft), the frequency equation of the fixed-free cantilever beam was solved for EI assuming a nominal frequency of vibration of 2.00 Hz. This resulted in an initial estimate of $EI = 3.18 \times 10^{13}$ lb/ft.^{2} Values of k_1 and k_2 were then found so that the natural frequency and mode shape of the model would match those measured in the first test. A slight adjustment in the initial estimate of EI was also required. Next, EI was held constant and the values of k_1 and k_2 were adjusted until the foundation translation and rotation reported by Foutch were obtained.

The results are shown in Figure 4. The slight adjustment in EI so that Kuroiwa's results could be matched resulted in a fixed-base frequency of 2.02 Hz. The frequency of vibration that was then obtained after adjusting k_1 and k_2 until the predicted mode shape matched that of the second test was 1.65 Hz. This result is in good agreement with the value of 1.76 Hz that was measured by Foutch (1976), considering the simplicity of the model. The calculated frequency is about 6 per cent lower than the measured value.

The analysis of the Millikan Library building presented above was based on an Euler beam model of the structure with springs included at the base to simulate the effects of soil-structure interaction. The results of the analysis suggest that the change in the natural frequency of vibration of the structure determined by forced vibration tests conducted before and after the San Fernando earthquake are consistent with the observed changes in the mode shape of the building. However, only bending deformations in the structure were assumed.

An additional analysis was conducted to determine if the results are sensitive to the model employed. The model chosen for the additional investigation was a lumped-mass model schematically shown in Figure 5. This model is more representative of the actual structure since shearing deformation in the walls is included in the analysis and the mass distribution is modeled more realistically. A simple analysis indicated that approximately 20 to 25 per cent of the roof deflection is due to shearing deformations in the walls.

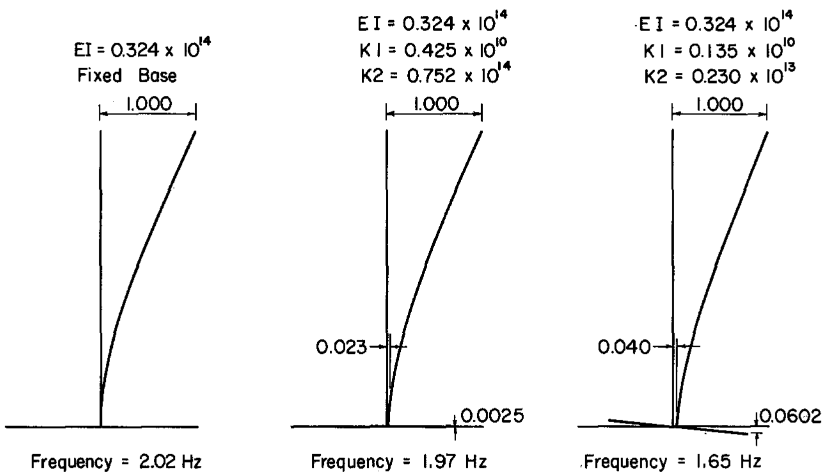
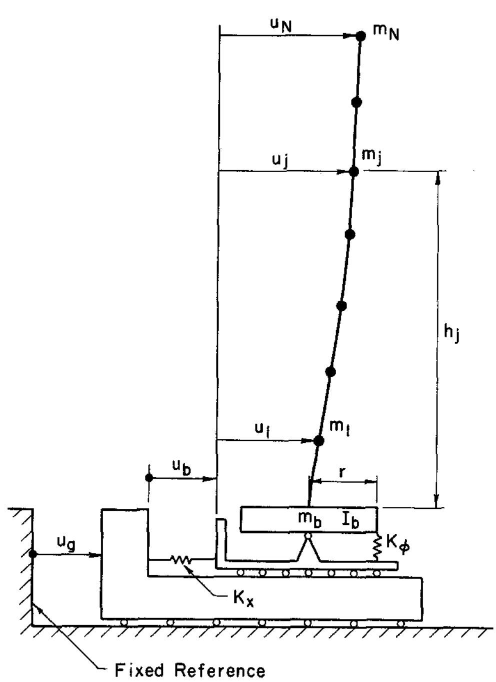


FIG. 4. Results of analysis using an Euler beam model of Millikan Library: (a) fixed base model; (b) rotational and translational springs added to base to match mode shape and frequency determined by Kuroiwa; (c) base springs adjusted to match mode shape measured by Foutch.



SOIL-STRUCTURE INTERACTION MODEL

FIG. 5. Lumped-mass model.

The equations of motion for the undamped system shown in Figure 5 may be written

$$M\underline{\ddot{u}}^s + K\underline{u}^s + M(h\ddot{\varphi} + \ddot{u}_g + \ddot{u}_b) = 0$$

$$\sum_{j=1}^{n} m_j(\ddot{u}_j^s + h_j\ddot{\varphi} + \ddot{u}_g + \ddot{u}_b) + m_b(\ddot{u}_g + \ddot{u}_b) + K_x u_b = 0$$

$$\sum_{j=1}^{n} m_j h_j(\ddot{u}_j + h_j\ddot{\varphi} + \ddot{u}_g + \ddot{u}_b) + I_t\ddot{\varphi} + K_\varphi r^2\varphi = 0, \tag{6}$$

or in matrix form

$$M^*\underline{\ddot{u}} + K^*\underline{\ddot{u}} = -\underline{m}\ddot{u}_g \tag{7}$$

where

u_g = acceleration of the ground
u_b = the relative translation between the base of the structure and the fixed reference
$\underline{u}^s$ = a vector of horizontal displacements of the floors relative to the base
u_j^s = the horizontal displacement of floor j relative to the base
$\underline{h}$ = a vector of floor heights above the base
h_j = the height of floor j above the base
φ = the rotation of the base
I_t = the sum of the centroidal moments of inertia of the floors and base masses
K_φ = the fixed base stiffness matrix of the model
K_x = the foundation rotational spring constant
M = the fixed base mass matrix of the structure.

The flexibility matrix was determined for the fixed base model by applying a unit load at each floor level and computing the resulting displacements at all floor levels. This was accomplished by using the plane stress solution (Housner and Vreeland, 1966) for the displacement of the centerline of a rectangular cantilever beam with a concentrated load applied at its end. The resulting flexibility matrix was then inverted to obtain the fixed base stiffness matrix, K, of equation (6). The masses of the various floors as estimated by Kuroiwa were used to form the mass matrix.

From this point, the procedure was identical to that described for the previous model. Values of EI, K_x, and K_φ were then adjusted until the mode shape measured after the earthquake was obtained for the model. The resulting natural frequency could then be compared to the measured post-earthquake value. Results of the analysis are shown in Figure 6.

Discussion

It is seen in the figure that the final natural frequency that resulted when the mode shape that matched the present condition of the building was obtained was 1.68 Hz. This is within 5 per cent of the measured value of 1.76 Hz.

The results of this analysis and that for the previous model indicate that the observed changes in the natural frequency of vibration of the building are compatible with the measured changes in the foundation rotation and translation. This implies that experimental error is not required to explain the differences in the mode shape measured in the two tests. It seems most probable to the authors that a significant change in the behavior of the foundation in the N-S direction did, in fact, occur during the San Fernando earthquake. If this is the case, a reduction by a factor of about 33 in the effective rotational stiffness of the foundation is required. Although this appears to be a large change, in the application of simple concepts of soil-structure interaction in which the foundation is idealized as circular, this may be associated with a change in the effective radius of the building foundation of about 1.7 meters. Also, it should be noted that the reduction in natural frequency is relatively insensitive to changes in the rotational stiffness. This is clarified by the realization that changing the foundation model from a fixed-base condition to that measured by Kuroiwa represents an infinite decrease in the stiffness of the base springs with only a 2 per cent change in the frequency.

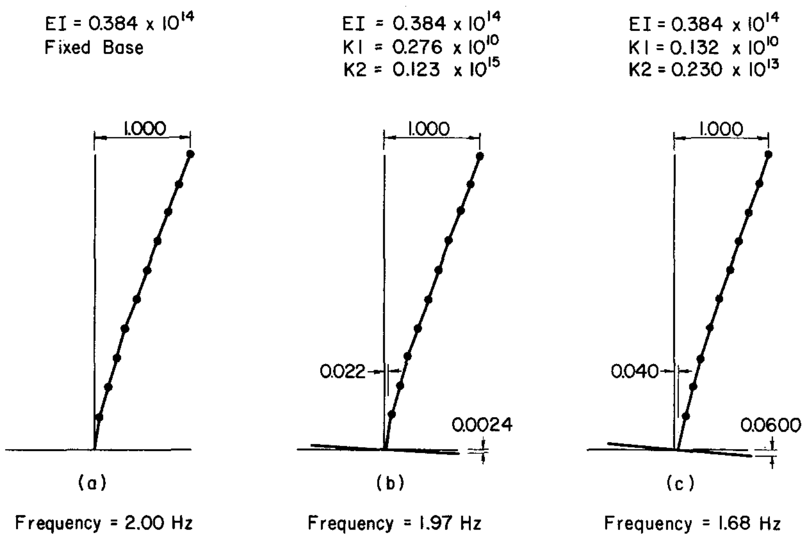


FIG. 6. Results of analysis using a lumped-mass model of Millikan Library: (a) fixed base model; (b) rotational and translational springs added to base to match mode shape and frequency determined by Kuroiwa; (c) base springs adjusted to match mode shape measured by Foutch.

A detailed examination of the foundation of Millikan Library was made to see if any obvious sources of stiffness reduction could be found. The results are shown in Figure 7 which is a plan view of the ground, or plaza, level of the structure. The dark lines indicate the structural walls and columns, including the one-story board room to the East. The lighter lines show the outline of the concrete slabs at the plaza level which is about 1 meter above grade. Also shown are appurtenant walkways, retaining walls, and ramp structures.

As shown in the figure, there is a variety of damage on the plaza level consisting of spalling of concrete and cracks with widths varying from 0.5 to 2 mm. The larger cracks and the spalling are thought to have occurred during the San Fernando earthquake; but one cannot be sure. The most significant damage that might contribute to a reduction of rotational stiffness is on the north side of the building near the west end. The retaining wall for the ramp structure has been pushed about 3 mm away from the building to the North and about 6 mm down, and the top of the wall has failed in compression. There is no expansion joint at this juncture. It is estimated that the maximum relative N-S motion at this level was about 4 mm during the earthquake. There is a similar wall on the south side of the building, but where this wall joins the building there is an expansion joint with about 12 mm of filler material.

The joint has opened about 3 mm, but there is no obvious sign of compression failure of the concrete. The larger cracks on the west end of the plaza are also consistent with reduced rotational stiffness in both the N-S and E-W directions. The crack on the north side of the west end is particularly prominent and continues vertically about 300 mm up into the granite facing of the building at the corner of the west shear wall. The vertical displacement on this crack varies from about 8 mm at the north end to zero at the south end of the crack, as indicated in Figure 7. The other cracking and

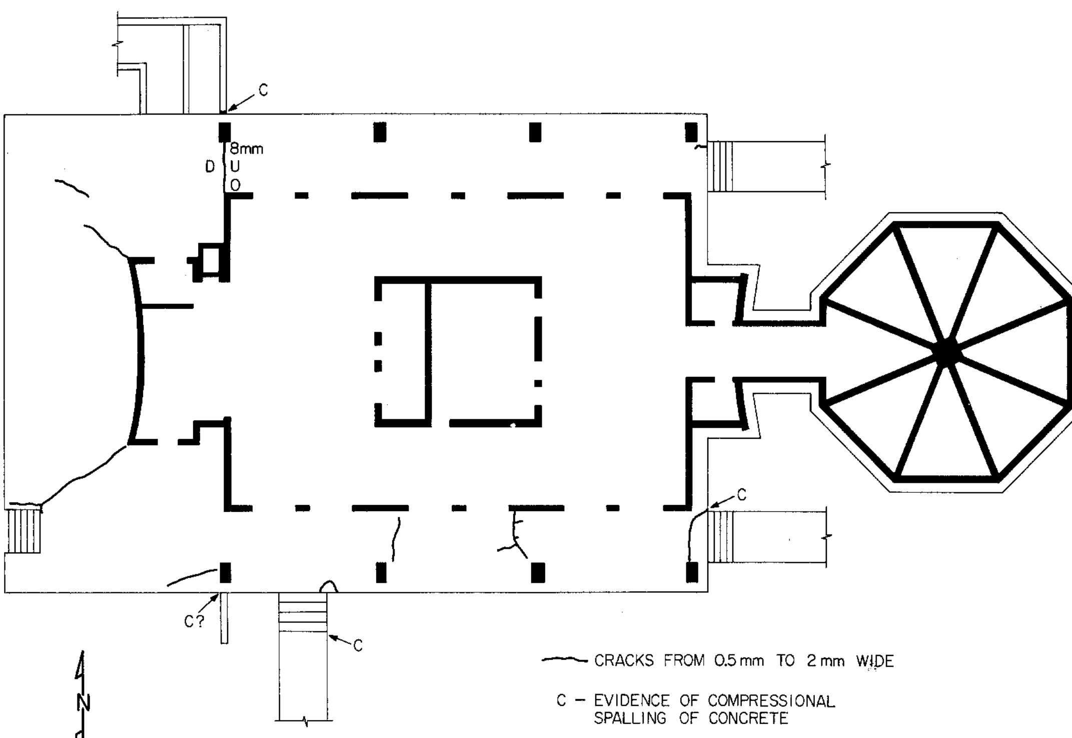


FIG. 7. Plan view of ground level of Millikan Library showing locations of spalling of concrete and major cracks.

spalling noted in the figure is also consistent with loss of rotational, and in some cases translational, stiffness.

At the foundation level, one level below that shown in Figure 7, a steam tunnel enters the structure from the west near the northwest corner. There is a $\frac{1}{2}$-mm crack completely around the tunnel at the point where the tunnel meets the basement wall. It is, of course, conjectural whether this crack was caused by the earthquake, but it would reduce the small amplitude stiffness provided by the tunnel.

It is also possible that the earthquake response of the building has consolidated the soil under the foundation, particularly at the extremes of the structure where the dynamic stresses would be largest. The small amplitude stress conditions, and therefore the stiffness under small amplitudes, could be quite different from that which existed before the earthquake. A second point of interest concerning the soil is that simple theoretical estimates of the foundation stiffness using elastic half-space properties for the soil, modified for embedment, produce results which are consistent with the amount of interaction shown in the second tests (Foutch, 1976). This is consistent with the possibility that in the first test stiff, brittle elements were contributing to the foundation stiffness in addition to the soil.

Conclusions

Although experimental error cannot be completely ruled out, the authors believe the most probable cause of the difference in the two sets of test results to be the loss of the rotational and translational stiffness provided by the retaining walls, sidewalks, and concrete slabs at the ground floor. These stiff, but brittle elements could have governed the apparent foundation stiffness when the building was new, but are no longer effective. The possible contribution of such elements to increased building stiffnesses should be investigated in future vibration tests of full-scale structures.

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